EXTREMA IN ONE DIMENSION

Chapter 8 of "A MATHEMATICS COURSE FOR POLITICAL AND SOCIAL RESEARCH"

1 FIND ALL EXTREMA

a) $f(x) = x^3 - x + 1, \; x \in [0,1]$

$f'(x) = 3x^2 - 1 = 0 \Rightarrow 3x^2 = 1 \Rightarrow x^2 = 1/3$

$x = \pm\sqrt{1/3} \Rightarrow x^* = \frac{1}{3}$

$f''(x) = 6x \Rightarrow f'(1/3) = 2 > 0$, minimum

$f(1/3) = \frac{1}{27} - \frac{1}{3} + 1 = \frac{19}{27} \rightarrow$ global minimum $\quad f(0) = 1, \; f(1) = 1 \rightarrow$ global maximum

b) $f(x) = x^2 - 2x + 17, \; x \in [0,5]$

$f'(x) = 2x - 2 = 0 \Rightarrow x^* = 1$

$f''(x) = 2 > 0$, minimum

$f(2) = 1 - 2 + 17 = 16$ global minimum $\quad f(0) = 17$ máximo local (à direita) $\quad f(5) = 25 - 10 + 17 = 32$ global maximum

d) $f(x) = 2 - 3x, \; x \in [-3,10]$

$f'(x) = -3 \Rightarrow f$ é estritamente crescente

máximo $= f(-3) = 11$, mínimo $= f(10) = -28$ (globais)

e) $f(x) = 6x - x^2 + 12, \; x \in [0,10]$

$f'(x) = 6 - 2x \Rightarrow x^* = 3$

$f''(x) = -2 < 0$, máximo

$f(3) = 18 - 9 + 12 = 21 \rightarrow$ máximo global

$f(0) = 12$

$f(10) = 60 - 100 + 12 = -28 \rightarrow$ mínimo global

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$U(x) = -a(x - x_0)^2$, where x_0 represents the ideal location of the legislator and x represents the location of the current policy in the one-dimensional policy space. Prove that the legislator's utility is maximized when the policy is located at $x = x_0$

$U'(x) = -2a(x - x_0) \cdot (1)$ ~ pela regra da cadeia

$-2a(x - x_0) = 0 \Rightarrow x = x_0$

$U''(x) = -2a \Rightarrow$ maximum, com $a > 0$